A registration method using downhill simplex method and its applications

Akio Doi[1] and Tomohiro Chiba[1]
[1] Iwate Prefectural University,
Sugo 152-52, Takizawa-shi, Iwate-ken, Japan
doia@iwate-pu.ac.jp

Abstract. In this paper, we proposed an effective method for time series CT image registration using Mutual Information (MI) and downhill simplex method. By using this method it was possible to obtain correct results in a shorter time. When we include all range of histogram in MI evaluation, the value of MI becomes worse. In order to prevent this problem, we determine an effective range of image histogram. Moreover, in order to speed up the convergence, we propose an effective initial position specification and geometry based finish condition (vector terminate). In the experiment, 3D rigid body alignment is performed successfully by six variables of movement and rotation.

1 Introduction

As a part of medical image diagnosis, comparison studies are conducted in which images captured at different times and images obtained with different modalities are combined and observed. When these alignments are conducted by using an interactive operation, the inconsistency has increased, and it is difficult to compare three-dimensional images, such as CT (Computational Tomography). The comparison between different diagnostic devices, such as X-ray CT, MRI (Magnetic Resonance Imaging), and Ultrasound machines, is also a difficult problem.

Two important factors are generally considered in automatic image registration problems, which are "coincidence scale" and "an optimization method". In this paper, we focused on mutual information (MI) quantity as “coincidence scale”. The MI of two random variables is a measure of the mutual dependence between the two variables. Therefore, it is possible to compare different kind of variables, such as X-ray CT and MRI [1-3].

As “an optimization method”, we have utilized downhill simplex method in multi-dimensions. The downhill simplex method was introduced by Nelder and Mead [4]. The method makes use of simplex to find its way “downhill” to a minimum. A simplex in N dimensions consist of N+1 points or vertices and the lines that are connected these vertices. The merits of the downhill simplex method requires only function evaluations (derivative-free optimization), and it is easy to implement the programs. However, this method is “hill-climbing” method and therefore does not guarantee that a global minimum will be fund. Therefore, it is important to specify appropriate initial and terminate conditions in automatic image registration problems [5].

In this paper, we proposed an effective method for time series CT images registration using Mutual Information (MI) and downhill simplex method. The purpose of the registration was to investigate changes in bone shape change by time series. Therefore,

we assumed that the target and reference images are time series CT images of the same patient, and several improvements were made in order to align time series CT images quickly.
In general, MI for image registration was calculated by using all range of histogram. However, when we include all range of histogram in MI evaluation, the value of MI becomes worse due to the influence of other organs and air. In order to prevent this problem, we determine an effective range of image histogram. Moreover, in order to speed up the convergence, we propose both an effective initial position specification and geometry based finish condition (vector terminate). In the experiment, 3D rigid body alignment is performed successfully by six variables of movement and rotation.

2 Previous works

In an automatic image registration, the degree of matching is evaluated from the relative positional relationship of the image pair. There are several methods for expressing the degree of coincidence by quantitative numerical values, but in the registration between different modality images in particular, mutual information (Mutual Information) using the idea of entropy in the communication field is often used [1]. This is calculated from the probability distribution using the two-dimensional histogram composed of the image density information of the overlapped portion. The following equation is defined when the two-dimensional histogram is h (a_i, b_j) and the width of the histogram is bin.

$$\mathrm{MI(A,B)} = \sum_{i=1}^{bin}\sum_{i=1}^{bin} p(a_i, b_j)\, log_{\,2} \frac{p(a_i, b_j)}{p(a_i)p(b_j)} \tag{1}$$

Using the coincidence degree numerical expression such as MI, it is possible to define the evaluation value as a function of the movement amount, and finding the optimum solution of the function obtains the image matching position.
Next, an optimization method is used to obtain a solution, and one of the algorithms is the downhill simplex method [4]. This method must be started not just with a single point, but with N+1 points, defining an initial simplex. The method moves the point of the simplex where the function is largest through the opposite face of the simplex to a lower point. The calculation is continued until the move become smaller. These steps are called “reflection”, “reflection and expansion”, “contraction”, and “multiple contractions”. Assuming that the initial point is P_0, the other n points are

$$P_i = P_0 + \lambda e_i \tag{2}$$

Where the e_i are N unit vectors and λ is a constant representing the measure of the characteristic length of the problem. Although this method is used without requiring calculation of partial derivatives etc., it is often used for automatic registration of images because the algorithm is intuitive and easy to understand [5].

3 An Image Registration Using Downhill Simplex Method

In this chapter, we propose an initial position calculation method (Chapter 3, Section 1), an evaluation method for Mutual Information with a limited range (Chapter 3, Section 2), and a terminate condition (Chapter 3, Section 3) in order to improve the convergence of image registration using the downhill simplex method.
In the implementation, three-dimensional rigid body alignment was assumed, and the optimization process was performed by a total of six movement variables (movement(T_x, T_y, T_z) and rotation(Θ_x, Θ_y, Θ_z)).

3.1 Initial Position Calculation

The image matching degree shows a unimodal characteristic toward the matching position around the true matching position, but when it is too far from this position, a local solution is born and a correct optimum solution cannot be obtained by the downhill simplex method. Therefore, it is necessary to set the initial position in the vicinity of the matching position to some extent. Here, in order to find the best initial position adjustment, a 5 × 5 × 5 Laplacian filter is applied to each of the images, and the luminance centroid is obtained from the image. The Laplacian filter can extract the edges, which represent bone boundary, and the coordinates of the luminance center of gravity is efficient in comparison with coordinates of image center alignment. When a image resolution is $w \times h \times d$, and the value of the luminance is B_{ijk} in voxel coordinates(i, j, k), the luminance center of gravity $\left(g_i, g_j, g_k\right)$ can be calculated in the following equations:

$$g_i = \frac{\sum_{k=1}^{d} \sum_{j=1}^{h} \sum_{i=1}^{w} \left(w \cdot B_{ijk}\right)}{\sum_{k=1}^{d} \sum_{j=1}^{h} \sum_{i=1}^{w} B_{ijk}} \tag{3}$$

$$g_j = \frac{\sum_{k=1}^{d} \sum_{j=1}^{h} \sum_{i=1}^{w} \left(h \cdot B_{ijk}\right)}{\sum_{k=1}^{d} \sum_{j=1}^{h} \sum_{i=1}^{w} B_{ijk}} \tag{4}$$

$$g_k = \frac{\sum_{k=1}^{d} \sum_{j=1}^{h} \sum_{i=1}^{w} \left(d \cdot B_{ijk}\right)}{\sum_{k=1}^{d} \sum_{j=1}^{h} \sum_{i=1}^{w} B_{ijk}} \tag{5}$$

Fig. 1 shows the target image (left image) and the edge detection image (right image). Fig. 2 shows the reference (moving) image (left image) and the edge detection image (right image). The edge detection algorithm was 5 x 5 x 5 pixel Laplacian filter. In the 3 x 3 x 3 Laplacian filter, many discontinuous edges were left too much and good results were not obtained. The Red points in Figs. 1 and 2 represent the luminance center of gravity alignment. Fig. 3 shows the difference between image center and the luminance center of gravity alignments. In the first simplex definition, in order to prevent convergence to local solution, the evaluation value of ± λ is calculated for each dimension, and the one with the better evaluation value is taken as the vertex.

$$P_i = P_0 \pm \lambda e_i \quad \textbf{(6)}$$

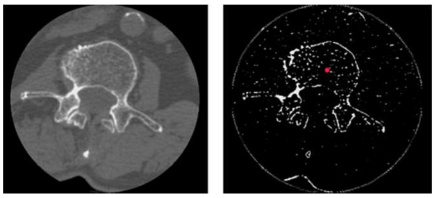

Fig. 1 Target Image and Edge Image

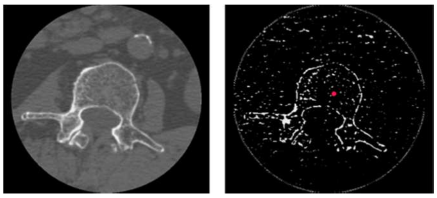

Fig. 2 Reference Image and Edge Image

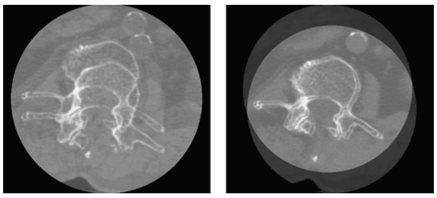

Fig. 3 Registration Result by Image Center and Gravity

3.2 An Evaluation Method for Mutual Information with a Limited Range

When we include all range of histogram in MI evaluation, the value of MI becomes worse. In order to prevent this problem, we find an effective range of the histogram in discriminant analysis. The mutual information quantity calculates the dependency of the histogram of the overlapping part of the two images. For example, if you use the image as it is, it will be affected by the background and air. Therefore, it is necessary to optimize the calculation method of the evaluation function so as to calculate the similarity of the bone shape. Strictly speaking, it is effective to adjust the target and reference images with the optimum window level and window. However, this operation takes time and it is difficult to find optimal histogram range for each image interactively. For this reason, in this research, as a solution to this problem, we propose a method to limit the range of the histogram used when computing mutual information as follows.

Step 1:

Using the discriminant analysis method, a threshold T1 obtained, and a background part and others are divided.

Step 2:

Find the histogram value T2 that appears first from the threshold T1 obtained in Step 1.

Step 3:

Set the value of T2 as the lowest value and the maximum value T3 of the histogram as the effective range to be used for mutual information quantity calculation.

3.3 MI Computation and Convergence Condition

Mutual information amount and each movement parameter dx, dy, dz, θx, θy, θz are all calculated as real numbers. Here, the luminance value of the target image overlapping with the moving image after coordinate conversion is obtained by triple linear interpolation. Trilinear interpolation is a method of multivariate interpolation on a three dimensional regular grid. It is often used to interpolate within cells of a volumetric data set. Since the voxel size of CT images is not unit size, simple translation and scale (each axis independently) can be used.

Trilinear interpolation is a method of obtaining a value corresponding to the distance by using the luminance value of 8 pixels around the coordinates after the coordinate transformation. Let f (x, y, z) be the luminance value at the coordinates (x, y, z) (u, v, w), (u + 1, v, w), (u, v + 1), the coordinates after the coordinate transformation are (x, y, z) (u, v + 1, w + 1), (u, v + 1, w + 1), (x, y, z) of the target image at the coordinates (x, y, z) can be obtained by the following expression, where the luminance value f (x, y, z) is (u + 1, v + 1, w + 1) Figure 4.9 shows the distance relationship between the coordinates after coordinate transformation and the surrounding eight pixels.

$$\begin{aligned} f(x,\ y,z) = {} & (1-\alpha)(1-\beta)(1-\gamma)f(u,v,w) \\ & + \alpha(1-\beta)(1-\gamma)f(u+1,v,w) \\ & + (1-\alpha)\beta(1-\gamma)f(u,v+1,w) \\ & + \alpha\beta(1-\gamma)f(u+1,v+1,w) \\ & + (1-\alpha)(1-\beta)\gamma f(u,v,w+1) \\ & + \alpha(1-\beta)\gamma f(u+1,v,w+1) \\ & + (1-\alpha)\beta\gamma f(u,v+1,w+1) \\ & + \alpha\beta\gamma f(u+1,v+1,w+1) \end{aligned} \quad \textbf{(7)}$$

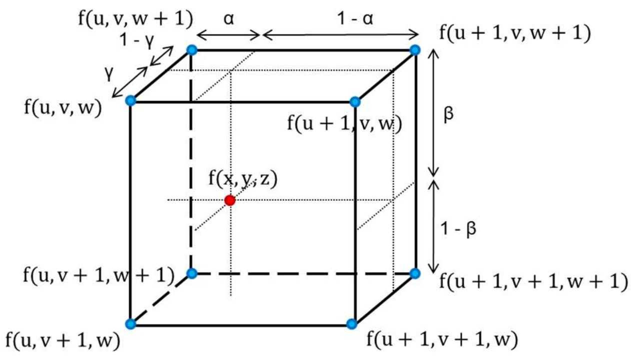

Fig. 4 Trilinear Interpolation

In the latter half of the search, the coordinate movement of the simplex vertex is extremely small, and it becomes almost meaningless on the image registration. In general, termination criteria can be delicate in any multidimensional minimization problems. We can check the decrease in the function value in the terminating step becomes smaller than some tolerance ftol. In this case, the function value of the best / worst point of simplex vertex is used for the termination condition of the downhill simplex method.

$$\frac{2|f(x_h) - f(x_i)|}{|f(x_h)| + |f(x_i)|} < ftol \qquad \textbf{(8)}$$

Here, $f(x_h)$, $f(x_i)$ is function values of best and worst points of simplex vertex, respectively. The user defined number ftol is some tolerance.Alternatively, it is possible to terminate when the vector distance become smaller than some tolerance ftol.

$$\sqrt{\begin{array}{c}(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2 \\ +(\theta_{xi} - \theta_{xj})^2 + (\theta_{yi} - \theta_{yj})^2 + (\theta_{zi} - \theta_{zj})^2\end{array}} \quad < \quad \mathrm{ftol} \qquad \textbf{(9)}$$

Here, (x_j, y_j, z_j, θ_{xj}, θ_{yj}, θ_{zj}) and (x_i, y_i, z_i, θ_{xi}, θ_{yi}, θ_{zi}) are the coordinates of the best point, the coordinates of the worst point, respectively. The user defined number ftol is some tolerance. In chapter 4, we compared function value and vector distance criteria as a terminating condition, and we show the condition of Eq. (9) by using vector distance (the coordinate distance of best and worst points) is faster than function value criteria.

4 Experimental Results

Using the image pairs of 4 patient cases, CT images (spirally captured images taken at different times), we evaluated two approach: (1) Method-A (function value criteria): MI computation for all range of histogram, and function value criteria", and (2) Method-B: MI computation for restricted range of histogram, and distance vector criteria". In the method-B, we used Eq. (9), which shows a distance evaluation between parameters (x_j, y_j, z_j, θ_{xj}, θ_{yj}, θ_{zj}) as the convergence condition and we gave ftol as 0.1 in the experiment. Both methods were carried out from the initial position obtained by Chapter 3 Section 1. The computational environment was Intel Core i7-4790 CPU (3.60GHz) and Main Memory 8.00GB. Table 1 summarizes the results.

Table 1 Visual Evaluation and Computational Time

	① Method A			② Method B		
Case	MI	Visual Eval.	Time(s)	MI	Visual Eval.	Time(s)
1	1.11652	○	1089	1.20741	○	600
2	0.82979	×	1491	0.98517	○	696
3	1.70564	○	1364	1.48884	○	826
4	1.02256	∆	1083	1.16263	○	767

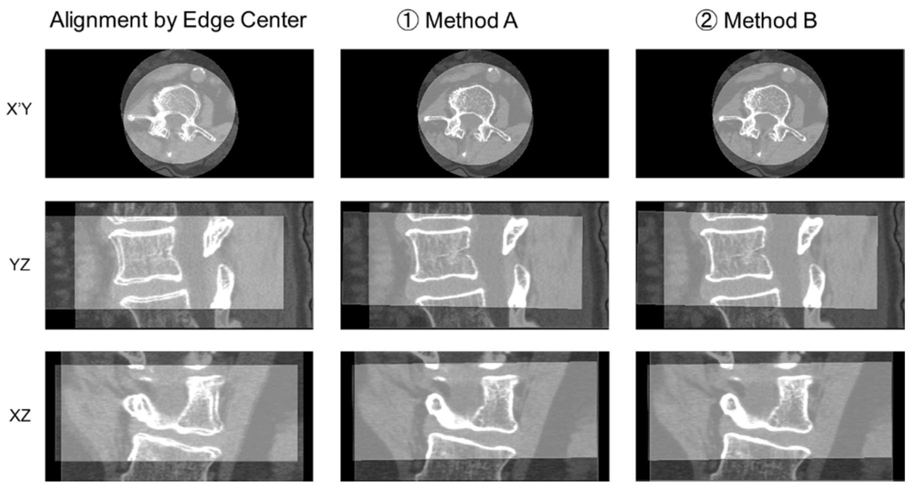

Fig. 5 Registration Results of Case 1(Left, Center, Right: registration by using the luminance center of gravity, Method-A, Method-B, respectively)

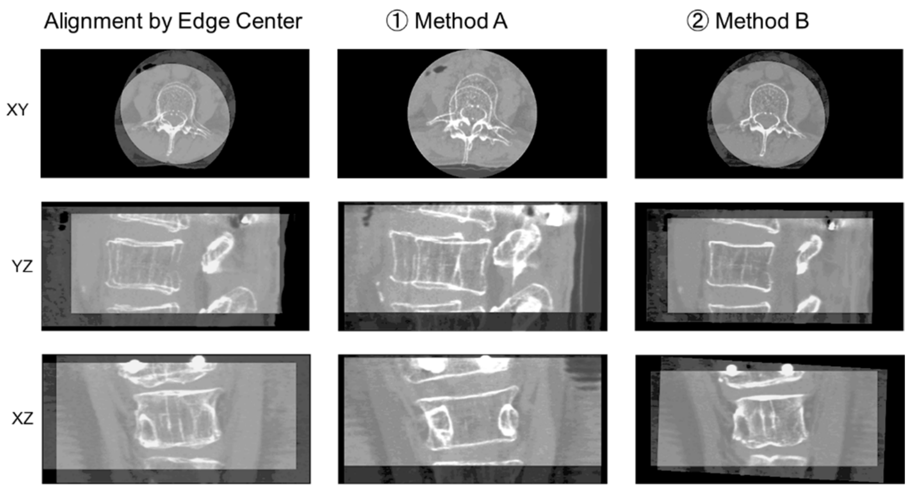

Fig. 6 Registration Results of Case 2(Left, Center, Right: registration by using the luminance center of gravity, Method-A, Method-B, respectively)

4 Conclusion

In this paper, we proposed an effective method for time series CT image registration using Mutual Information (MI) and downhill simplex method. By using this method it was possible to obtain correct results in a shorter time. In our approach, we determined an effective range of image histogram in order to converge into the optimal position. Moreover, in order to speed up the convergence, we proposed an effective initial position specification and geometry based finish condition (vector terminate). The experiments were performed on four cases of pair of time series CT images, and the proposed method was able to improve alignment accuracy in comparison with the ordinary methods. In the experiment, 3D rigid body alignment is performed successfully by six variables of movement and rotation. Though we performed these experiments by using a downhill simplex method, we need the experiments with another optimize algorithm, such as Powell method [6]. We expect a little speed-up in comparison with a downhill simplex method.

Acknowledgments.
For this research, we received research grants from JSPS Grant-in-Aid for Research 26350541 (Foundation C) "Fundamental research on preoperative and intraoperative support system for tailor made medicine". Part of this research was conducted using the Iwate Prefectural University Academic Research Fund. This research was partially supported by “Ring! Ring! (KEIRIN & AUTORACE)” project, which is bicycle racing & auto racing assistant research aid: “http://hojo.keirin-autorace.or.jp/”. We also received a research grant from the Ministry of Economy, Trade and Industry Medical Industry Cooperation Project Development Project "Development of Innovative Spinal Braking Implant Using 3D Titanium Printer".

References

1. Joisen P. W. Pluim, J. B. Antoine Maintz and Max A. Viergever: Mutual information based registration of medical images: a survey, IEEE Trans. on Medical Imaging, Vol. 22, No. 8, (2003), 986-1004.
2. Witold Kosinski, Pawel Michalak, Piotr Gut: Robust Image Registration Based on Mutual Information Measure, Journal of Signal and Information Processing, 3 (2012), 175-178.
3. Hemant D. Tagare and Murali Rao: Why Does Mutual-Information Work for Image Registration? A Deterministic Explanation, IEEE Trans. on Pattern Analysis and Machine Intelligence, Vol. 37, No. 6 (2015) 1286-1296.
4. Nelder, John A.; R. Mead.: A simplex method for function minimization”, Computer Journal, 7 (1965), 308-313.
5. William H. P, Saul A. T, William T. V, Brain P. F.: Numerical Recipes in C The Art of Scientific Computing Second Edition, (2010), 408-420.
6. Powell, Michael J. D.: On Search Directions for Minimization Algorithms, Mathematical Programming, 4 (1973) 193–201.